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The induced electric field due to a current transient

Y Beck, A Braunstein and S Frankental

Department of Interdisciplinary Studies, Faculty of Engineering, University of Tel-Aviv, PO Box 39040, Tel-Aviv 69978, Israel

E-mail: beck@post.tau.ac.il

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Abstract

Calculations and measurements of the electric fields, induced by a lightning strike, are important for understanding the phenomenon and developing effective protection systems. In this paper, a novel approach to the calculation of the electric fields due to lightning strikes, using a relativistic approach, is presented. This approach is based on a known current wave-pair model, representing the lightning current wave. The model presented is one that describes the lightning current wave, either at the first stage of the descending charge wave from the cloud or at the later stage of the return stroke. The electric fields computed are cylindrically symmetric. A simplified method for the calculation of the electric field is achieved by using special relativity theory and relativistic considerations. The proposed approach, described in this paper, is based on simple expressions (by applying Coulomb's law) compared with much more complicated partial differential equations based on Maxwell's equations. A straight forward method of calculating the electric field due to a lightning strike, modelled as a negative–positive (NP) wave-pair, is determined by using the special relativity theory in order to calculate the 'velocity field' and relativistic concepts for calculating the 'acceleration field'. These fields are the basic elements required for calculating the total field resulting from the current wave-pair model. Moreover, a modified simpler method using sub models is represented. The sub-models are filaments of either static charges or charges at constant velocity only. Combining these simple sub-models yields the total wave-pair model. The results fully agree with that obtained by solving Maxwell's equations for the discussed problem.

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1. Introduction

A lightning strike is a natural electric phenomenon, which produces an electromagnetic field that expands spherically at the velocity of light. A direct lightning strike can be devastating and may injure human beings, and electrical and electronic equipment. Even an indirect lightning strike is dangerous due to the above-mentioned electromagnetic field. Over-voltages in elements of electronic and electrical systems and devices can be detected. These over-voltages may damage the equipment, disrupt magnetic memory devices, burn sensitive components and even destroy complete systems [1–5].

The calculations and measurements of the electric and magnetic fields, induced by the lightning strike, are important for understanding the phenomenon and developing effective protection systems. There are different approaches to address this problem, and extensive work has been done on the subject in the last 40 years. Analytical solutions for the electromagnetic fields based on Maxwell's equations were developed by different researchers. Uman and McLain [6] calculate the magnetic field due to various assumed forms of channel current as a function of time. Expressions for the far field in the time domain and computer solutions for the total near and far fields are presented in later research by Uman, McLain and Krider [7]. This work is based on an antenna model. Other techniques for calculating the electromagnetic fields due to lightning are the monopole and dipole techniques presented by Uman and Rubinstein [8] and Thottappillil and Rakov [9]. An interesting analysis of the electromagnetic field from a vertically placed and moving square wave and a wire carrying uniform line charge are discussed by Rubinstein and Uman [10] and later on by Thottappillil, Uman and Theethayi [11]. Time-domain analysis is important for deeper understanding of the transient phenomenon. Two approaches of the time-domain analysis of the electric fields from lightning return stroke are reviewed and compared by Thottappillil and Rakov [12]. One is the Lorentz condition approach and the other is the Continuity equation approach. A complementary effect to those discussed above is an important effect of retardation. The origin of this effect is the well-known Lienard–Weichert potential to a single point charge. This idea was developed to the so-called F-factor analysed for different cases by Thottappillil, Uman and Rakov [13].

Another calculation method is based on a wave-pair model presented by Braunstein [14]. This time-domain model describes the lightning wave as a step function, and the field is calculated directly from Maxwell's equations, with the addition of the so-called retarded potentials. Although this method gives an analytical result, it is complicated, requiring solving partial differential equations, and therefore not easy to apply.

In this paper, a novel method of calculating the induced electric fields is presented. The electric field due to a lightning strike is determined by using the Special Relativity Theory and relativistic concepts in order to calculate the 'velocity field' and the 'acceleration field' [15]. These fields are the basic elements required for calculating the total field resulting from the current wave-pair model. The results were compared with the results obtained by using Maxwell's equations. Comparison of the two methods yields identical results [16]. This approach yields to simple calculations and include inherently the retardation effects that are considered in the above-mentioned works. Therefore, the material presented supports the previous work based on classic electrodynamics yet introduces different and interesting new approaches.

2. Analytical solution: the direct approach

2.1. General

The calculation method presented in this paper is based on a model that describes the lightning current wave, either in the first stage of the descending of a charge wave from a cloud (figure 1) or at the later stage of the return stroke [17]. The source is defined as a semi-infinite thin charge filament (current wave) that is cylindrically symmetric [14]. This filament of charge

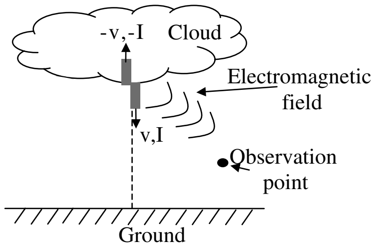

Figure 1. The descent of the current wave from the cloud.

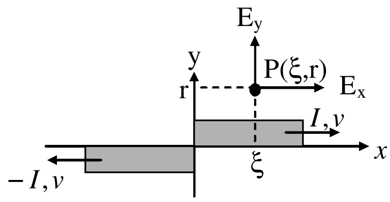

Figure 2. The opposite polarity two wave model.

is lowered from the cloud towards the ground by a down-going leader, causing electric field strength in the surrounding space. The field is propagating as a sphere from the front of the down-going current wave. The medium around the source is free space and the ground is assumed as the horizontal plane surface of infinite conductivity.

The model consists of two step-current waves starting at the origin that is the bottom of the cloud. The first one is a positive polarity wave travelling in the positive x-direction at a velocity of v. The second one is a negative polarity wave travelling in the $-x$ direction, also at a velocity of v, see figure 2.

Analysing the response of the system to a step function is justified by the fact that if the induced electric fields due to a step function are known, and then by using the Convolution Theory, the response to any other function form (i.e. a typical real lightning current wave) can be obtained. Moreover, the step response is easier to find and track and represents the physical descent of the charges at the discharge stage of the lightning generation phenomena.

Due to the fact that the source of the lightning current is not defined, an NP step wave-pair is used, as shown in figure 2, where the negative polarity current wave moving in the opposite direction is required in order to satisfy the boundary conditions of the problem and thus, to avoid the necessary definition of the source. (See the appendix.) This wave-pair model is with total agreement with the conservation of the charge and therefore this model is suitable in the case of the source of the lightning discharge.

The electric field is calculated at an observation point $P(\xi, r)$ as a function of time, t. The step function represents moving charges, where each charge is stationary at the origin prior to $t = 0$. At $t = 0$, the charges are infinitely accelerated to velocity v, and then continue to travel with a constant velocity v. The field of a charge moving at a constant velocity (‘the velocity field’) and the field of an accelerating charge (‘the acceleration field’) must be determined prior to the calculation of the electric field induced by the charges of the complete model.

2.2. *The velocity field*

A combination of Coulomb’s law and the special relativity theory is the basis for the calculation of the field due to a charge q travelling at a constant velocity v, see figure 3.

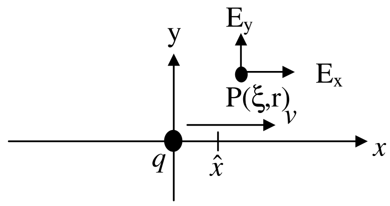

Figure 3. A charge moving at a constant velocity.

The electric field components in the x- and y-directions at the observation point $P(\xi, r)$, are

$$\left.\begin{aligned} E_{vx} &= \frac{q}{4\pi\varepsilon_0}\frac{\gamma\xi}{[(\gamma\xi)^2+r^2]^{3/2}}\cdot\delta_{-1}\left(t-\frac{\rho}{c}\right) \\ E_{vy} &= \frac{q}{4\pi\varepsilon_0}\frac{\gamma r}{[(\gamma\xi)^2+r^2]^{3/2}}\cdot\delta_{-1}\left(t-\frac{\rho}{c}\right) \end{aligned}\right\} \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1}$$

where ξ and r are the horizontal and vertical distances from the origin, $\rho = \sqrt{\xi^2+r^2}$ is the radial distance from the origin and γ is the special relativity factor. The calculation is in Cartesian coordinates due to the fact that only two-dimensional surface is considered. The above-mentioned expression is calculated, using the Lorentz transformation, for transforming the coordinates to a coordinate system, in which the source charge q is stationary. In the new coordinate system, the electric field is calculated by means of Coulomb's law. The calculated field is then transformed back to the original coordinate system by the reverse Lorentz transformation [15]. This expression is valid only when the information about the charge q at its constant velocity has reached the observation point $P(\xi, r)$. For an observer at that point, equation (1) yields the electric field strength only after the charge has passed point $\hat{x}$ on the x-axis. This means that

$$\left.\begin{aligned} &\frac{\hat{x}}{v} \geqslant \frac{1}{c}\sqrt{\xi^2+r^2} \quad \Rightarrow \quad \hat{x} \geqslant \beta\rho \\ &\text{where} \quad \beta = \frac{v}{c} \quad \text{and} \quad \rho = \sqrt{\xi^2+r^2} \end{aligned}\right\}. \tag{2}$$

2.3. *The electric field of a point charge accelerated to a constant velocity*

Consider the electric field lines of a stationary charge at the origin ($x = 0$) at $t = 0$. At $t = 0$ it is instantaneously accelerated to a constant velocity v. The principle field lines, in this case, are shown in figure 4.

Two types of electric fields can be observed in figure 4 [18], namely, the 'far field' and the 'near field'. Consider these two fields.[1]

(1) The 'far field' exists due to the fact that prior to the initial charge motion, it was stationary at the origin. The information about its position and movement travels at the velocity of light as an electromagnetic wave. The radiating electromagnetic wave has no preferences for the direction; therefore, it propagates in a sphere defined as SOI (sphere of influence). Thus, the electric field at a radius larger than ct must be identical to the one due to stationary charge at the origin.

[1] Note that the so-called 'near field: and 'far field' described here are not the same known fields in antenna theory/.

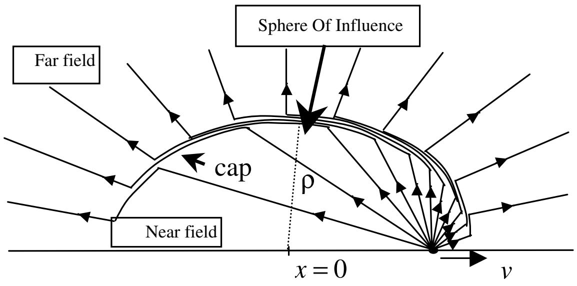

Figure 4. The field lines of a charge accelerated to a constant velocity v after being stationary at the origin.

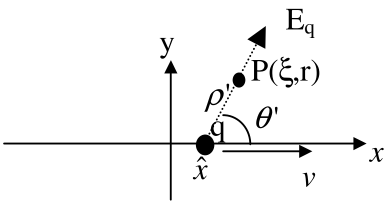

Figure 5. The electric field due to a charge travelling at a constant velocity.

(2) The 'near field' exists at a radius smaller than ct. Since the charge is moving at a constant velocity v, the electric field has a form as described in equation (1).

These two electric fields can be superimposed only by considering a field in a form of a spherical cap, expanding at the velocity of light, with its centre at the origin, $x = 0$, see figure 4. The spherical cap describes the electric field due to the acceleration of the charge.

The followings may be concluded about the acceleration field.

(1) The electric field on the surface of the cap is parallel to it.
(2) If the acceleration is not instantaneous, the width of the cap is proportional to the time of acceleration, i.e. between $v = 0$ and the constant velocity v. In the extreme infinite acceleration case (step function), the width of the cap will be zero, and the field will be an impulse function parallel to the surface of the cap.

In order to study the acceleration field, the expression for the field of a charge, moving at constant velocity, equation (1), should be rewritten in spherical coordinates. The origin at the point $\hat{x}$ represents the location of the wave front with a distance ρ' and an angle θ' to the observers point $P(\xi, r)$, as shown in figure 5.

Thus, the expressions obtained are

$$\vec{E}_{qv} = \frac{q}{4\pi\varepsilon_0}\frac{1}{\rho'}\frac{1-\beta^2}{[1-\beta^2\sin^2\theta']^{3/2}}\vec{i}_{\rho'} \qquad \text{for} \quad \rho < ct \tag{3}$$

where

$$\begin{cases} \rho' = \sqrt{(\xi - \hat{x})^2 + r^2} \\ \xi - \hat{x} = \rho'\cos\theta' \\ r = \rho'\sin\theta' \end{cases} \tag{4}$$

and $\vec{i}_{\rho'}$ is a unit vector in the ρ' direction.

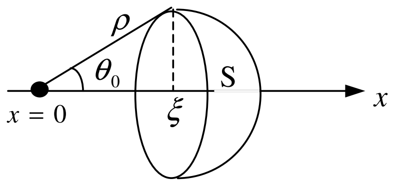

Figure 6. The spherical cap representing the emerging flux of the stationary charge.

If the charge begins its motion at the origin, $x = 0$, equation (3) represents then the field within a sphere with a radius ct centred at the origin. Outside this sphere, the field is identical to the field of a static point charge at the origin, $x = 0$. The equation describing this field in spherical coordinates is

$$\vec{E}_{qS} = \frac{q}{4\pi\varepsilon_0}\frac{1}{\rho^2}\vec{i}_\rho \qquad \rho > ct \tag{5}$$

where

$$\rho = \sqrt{\xi^2 + r^2}. \tag{6}$$

Rewriting the same equation in Cartesian coordinates equation (5) yields

$$\left.\begin{aligned} E_{xqS} &= \frac{q}{4\pi\varepsilon_0}\frac{\xi}{\rho^3}\delta_{-1}\left(\frac{\rho}{c} - t\right) \\ E_{yqS} &= \frac{q}{4\pi\varepsilon_0}\frac{r}{\rho^3}\delta_{-1}\left(\frac{\rho}{c} - t\right) \end{aligned}\right\}. \tag{7}$$

There is a discontinuity in the transition from the near field to the far field, at the sphere, whose radius is $\rho = ct$. Moreover, the flux penetrating any spherical cap from its interior region is not equal to the flux emerging from it into the exterior region. Since there is no source of flux in-between there must be a third flux on the sphere cap itself. This problem is discussed now.

Assume a spherical cap S, whose origin is at $x = 0$. The cap has a radius ρ and a polar angle θ_0, relative to the x-axis, see figure 6.

The exterior field is uniform and therefore, the emerging flux ϕ_1 from S, is the calculated static field, see equation (5), multiplied by the surface of the cap. This means that

$$\begin{aligned} \phi_1 &= \frac{q}{4\pi\varepsilon_0}\frac{1}{\rho^2}\int_0^{\theta_0} 2\pi\rho^2 \sin\theta' \,\mathrm{d}\theta' \\ &= \frac{q}{4\pi\varepsilon_0}2\pi[1 - \cos\theta_0]. \end{aligned} \tag{8}$$

A circle defines the basis of the cap $S(\rho, \xi)$, for which $\xi = \rho\cos\theta_0$. Thus equation (8) can be rewritten as

$$\phi_1 = \frac{q}{4\pi\varepsilon_0}2\pi\left[1 - \frac{\xi}{\rho}\right]. \tag{9}$$

The same cap, S, creates an angle ϕ_0 with the x-axis, which is the axis of the primed system (a system that travels together with the point charge at a constant velocity v). Since the interior field is radial, the flux penetrating S from the interior is equal to the flux which crosses a spherical cap S', centred at $x' = 0$ and subtending a polar angle ϕ_0, see figure 7.

The flux, in this case, is calculated in the same manner as above:

$$\begin{aligned} \phi_2 &= \frac{q}{4\pi\varepsilon_0}\frac{1}{\rho'^2}\int_0^{\varphi_0} 2\pi\rho'^2 \sin\theta \frac{1-\beta^2}{[1-\beta^2\sin^2\theta]^{3/2}}\,\mathrm{d}\theta \\ &= \frac{q}{4\pi\varepsilon_0}2\pi\left[1 - \frac{\cos\varphi_0}{\sqrt{1-\beta^2\sin^2\varphi_0}}\right]. \end{aligned} \tag{10}$$

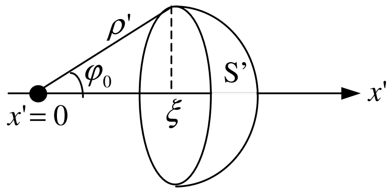

Figure 7. The spherical cap representing the incident flux of the moving charge.

The relations between the spherical and Cartesian coordinate systems are given in equation (4). Thus, equation (10) can be rewritten as

$$\phi_2 = \frac{q}{4\pi\varepsilon_0} 2\pi \left[1 - \frac{\xi - \hat{x}}{\sqrt{(\xi - \hat{x})^2 + (1 - \beta^2) r^2}} \right] \tag{11}$$

where x' is the distance the charge travelled from the origin, at a velocity v, at time t. As $\hat{x} = vt$ and $v = c\beta$, equation (11) takes the form

$$\phi_2 = \frac{q}{4\pi\varepsilon_0} 2\pi \left[1 - \frac{\xi - c\beta t}{\sqrt{(\xi - c\beta t)^2 + (1 - \beta^2) r^2}} \right]. \tag{12}$$

Since there are no charges near or at the discontinuity point (at the transition surface between the two types of fields), there is no other source that can add to the electric flux. The flux entering any finite (or infinitesimal) volume must be equal to the flux leaving the same volume. Due to this fact, the difference between the fluxes $\phi_1 - \phi_2$ must be compensated by a third flux emerging from the cap, but confined to the sphere surface. Due to the symmetry, it can be understood that this flux must be uniformly distributed along the circular edge of S, whose length is $2\pi r$. This requires an infinite electric field in the θ direction, which is confined to the surface of the sphere and still contributes a finite flux. Therefore, the flux has to be multiplied by a Dirac δ-function, which implies that the field is infinite but is located on the surface of the sphere. Thus, its integral along the edge of the spherical cap will produce the desired difference between the interior and exterior fluxes. The expression for this type of field is

$$\begin{aligned} \vec{E}_a &= \frac{\phi_1 - \phi_2}{2\pi r} \delta(\rho - ct) \vec{i}_{\theta_0} \\ &= \frac{q}{4\pi\varepsilon_0} \frac{1}{r} \left[\frac{\xi}{\rho} - \frac{\xi - c\beta t}{\sqrt{(\xi - c\beta t)^2 + (1 - \beta^2) r^2}} \right] \delta(\rho - ct) \vec{i}_{\theta_0}. \end{aligned} \tag{13}$$

In the presence of a δ function in equation (13), $c\beta t = \beta\rho$ and therefore

$$\vec{E}_a = \frac{q}{4\pi\varepsilon_0} \frac{\beta r}{\rho(\rho - \beta\xi)} \delta(\rho - ct) \vec{i}_{\theta_0}. \tag{14}$$

Equation (14) describes the field component due to a point charge that instantaneously accelerates from zero velocity to v, at $t = 0$.

In Cartesian coordinates, the unit vector $\vec{i}_{\theta_0}$ can be expressed as

$$\vec{i}_{\theta_0} = \frac{1}{\rho}[-\vec{i}_x r + \vec{i}_y \xi]. \tag{15}$$

Thus the 'acceleration field' can be rewritten as[2]

$$\left.\begin{aligned} E_{ax} &= -\frac{q}{4\pi\varepsilon_0}\frac{\beta r^2}{\rho^2(\rho-\beta\xi)}\delta(\rho-ct) \\ E_{ay} &= \frac{q}{4\pi\varepsilon_0}\frac{\beta r\xi}{\rho^2(\rho-\beta\xi)}\delta(\rho-ct) \end{aligned}\right\}. \tag{16}$$

2.4. *The total field of a point charge accelerating to a constant velocity*

The complete expression for the evolution of the electric field in time produced by the charge at the observation point $P(\xi, r)$, consists of the contribution of the three fields.

(1) The static electric field that hold until the sphere of influence of the accelerated charge has reached the observation point as described in equation (7).
(2) The 'acceleration field' produced by the infinite acceleration of the charge as described in equation (16).
(3) The 'velocity field' produced by the charge in its uniform motion described in equation (1).

Equation (17) expresses this in terms of the variable $\tau = t - t'$, when at time $t = t'$ the charge is accelerated from its stationary position to velocity $v = \beta c$.

$$\left.\begin{aligned} E_{xq} &= E_{sx} + E_{ax} + E_{vx} \\ &= \frac{q}{4\pi\varepsilon_0}\left[\frac{\xi}{\rho^3}\delta_{-1}\left(\frac{\rho}{c}-\tau\right) - \frac{\beta r^2}{\rho^2(\rho-\beta\xi)}\delta(\rho-c\tau) + \frac{\gamma\xi}{[(\gamma\xi)^2+r^2]^{3/2}}\cdot\delta_{-1}\left(\tau-\frac{\rho}{c}\right)\right] \\ E_{yq} &= E_{sy} + E_{ay} + E_{vy} \\ &= \frac{q}{4\pi\varepsilon_0}\left[\frac{r}{\rho^3}\delta_{-1}\left(\frac{\rho}{c}-\tau\right) + \frac{\beta r\xi}{\rho^2(\rho-\beta\xi)}\delta(\rho-c\tau) + \frac{\gamma r}{[(\gamma\xi)^2+r^2]^{3/2}}\cdot\delta_{-1}\left(\tau-\frac{\rho}{c}\right)\right] \end{aligned}\right\}. \tag{17}$$

2.5. *The total field of the NP wave-pair model*

The calculated total field at the observation point $P(\xi, r)$ due to an NP wave-pair will be the superposition of the electric field produced by a filamentary stream of positive charges (positive current wave), with a line charge density λ, and the electric field produces by a filamentary stream of negative charges (negative current wave), with a line charge density $-\lambda$, see figure 8.

Each current wave emerges from the origin at $t = 0$ with a velocity of $v = \beta c$, and forms a current of $\lambda\beta c$ which extends over $0 < x < c\,\beta t$. The expression for the electric field strength due to the positive current wave is, again, composed of three terms.

(1) A term that represents the static electric field strength that holds until the spherical cap of the acceleration field reaches the observation point,

[2] Note that this expression for the 'acceleration field' matches radiation field in [19].

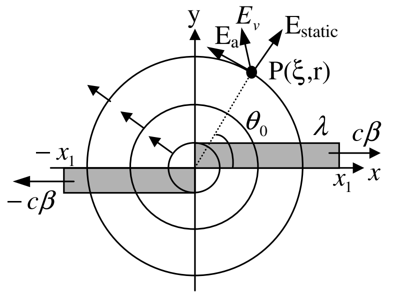

Figure 8. The static, velocity and accelerating fields of the NP wave-pair model.

$$\left.\begin{aligned} E_{Sx+} &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{\xi}{\rho^3}\cdot\int_0^t \beta c\delta_{-1}\left(\frac{\rho}{c}-\tau\right)\mathrm{d}\tau \\ &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{\xi}{\rho^3}\cdot\int_0^t \beta c\left[1-\delta_{-1}\left(\tau-\frac{\rho}{c}\right)\right]\mathrm{d}\tau \\ &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{\beta\xi}{\rho^2} \\ E_{Sy+} &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{r}{\rho^3}\cdot\int_0^t \beta c\delta_{-1}\left(\frac{\rho}{c}-\tau\right)\mathrm{d}\tau \\ &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{\beta r}{\rho^2} \end{aligned}\right\}. \tag{18}$$

(2) A term that represents the ‘acceleration field’:

$$\left.\begin{aligned} E_{ax+} &= -\frac{\lambda\beta c}{4\pi\varepsilon_0}\int_0^t \frac{\beta r^2}{\rho^2(\rho-\beta\xi)}\delta(\rho-c\tau)\,\mathrm{d}\tau \\ &= -\frac{\lambda}{4\pi\varepsilon_0}\frac{\beta^2 r^2}{\rho^2(\rho-\beta\xi)} \\ E_{ay+} &= \frac{\lambda\beta c}{4\pi\varepsilon_0}\int_0^t \frac{\beta r\xi}{\rho^2(\rho-\beta\xi)}\delta(\rho-c\tau)\,\mathrm{d}\tau \\ &= \frac{\lambda}{4\pi\varepsilon_0}\frac{\beta^2 r\xi}{\rho^2(\rho-\beta\xi)} \end{aligned}\right\}. \tag{19}$$

(3) A term that represents the ‘velocity field’ of the charge wave. It is better to integrate the expression over the distance $x = c\beta\tau$ rather the time τ, yielding

$$\left.\begin{aligned} E_{vx+} &= \frac{\lambda}{4\pi\varepsilon_0}\int_{\beta\rho}^{x_1}\mathrm{d}x\,\frac{\gamma(\xi-x)}{[\gamma^2(\xi-x)^2+r^2]^{3/2}} \\ &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\left\{\frac{1}{\gamma}\cdot\left[\frac{1}{[\gamma^2(\xi-x_1)^2+r^2]^{1/2}}\right]-\frac{1}{\gamma^2}\cdot\frac{1}{\rho-\beta\xi}\right\} \\ E_{vy+} &= \frac{\lambda}{4\pi\varepsilon_0}\int_{\beta\rho}^{x_1}\mathrm{d}x\,\frac{\gamma r}{[\gamma^2(\xi-x)^2+r^2]^{3/2}} \\ &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{1}{r}\left\{\frac{\gamma(\xi-X_1)}{[\gamma^2(\xi-x_1)^2+r^2]^{1/2}}-\frac{\beta\rho-\xi}{\rho-\beta\xi}\right\} \end{aligned}\right\}. \tag{20}$$

By the same manner, to calculate the electric field strength due to the negative current wave, the following expressions are obtained.

(1) Static field:

$$\left.\begin{aligned} E_{Sx-} &= -\frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{\beta\xi}{\rho^2} \\ E_{Sy-} &= -\frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{\beta r}{\rho^2} \end{aligned}\right\}. \tag{21}$$

(2) The 'acceleration field':

$$\left.\begin{aligned} E_{ax-} &= -\frac{\lambda}{4\pi\varepsilon_0}\frac{\beta^2 r^2}{\rho^2(\rho+\beta\xi)} \\ E_{ay-} &= \frac{\lambda}{4\pi\varepsilon_0}\frac{\beta^2 r\xi}{\rho^2(\rho+\beta\xi)} \end{aligned}\right\}. \tag{22}$$

(3) The 'velocity field':

$$\left.\begin{aligned} \vec{E}_{vx-} &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\left\{\frac{1}{\gamma}\cdot\left[\frac{1}{[\gamma^2(\xi+x_1)^2+r^2]^{1/2}}\right]-\frac{1}{\gamma^2}\cdot\frac{1}{\rho+\beta\xi}\right\} \\ \vec{E}_{vy-} &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{1}{r}\left\{\frac{1}{\gamma}\cdot\left[\frac{\gamma(\xi+X_1)}{[\gamma^2(\xi+x_1)^2+r^2]^{1/2}}\right]+\frac{\beta\rho+\xi}{\rho+\beta\xi}\right\} \end{aligned}\right\}. \tag{23}$$

The total field due to the positive and negative current waves is a net sum of all the terms of the different fields in the x-direction for obtaining E_x, and all the terms of the fields directed to the y-axis for obtaining E_y.

Note that the static field is redundant, as the contribution of a positive static charge, at the origin, negates the contribution of the negative one. Therefore, the total field is the sum of the velocity and acceleration fields only. These fields are defined by equations (19), (20), (22) and (23), thus the total field is

$$\left.\begin{aligned} E_x &= E_{vx}+E_{ax} \\ &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\left\{\frac{1}{\gamma}\cdot\left[\frac{1}{[\gamma^2(\xi-x_1)^2+r^2]^{1/2}}+\frac{1}{[\gamma^2(\xi+x_1)^2+r^2]^{1/2}}\right]-\frac{2}{\sqrt{\xi^2+r^2}}\right\} \\ E_y &= E_{vy}+E_{ay} \\ &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{1}{r}\left[\frac{\gamma(\xi-x_1)}{[\gamma^2(\xi-x_1)^2+r^2]^{1/2}}-\frac{\gamma(\xi+x_1)}{[\gamma^2(\xi+x_1)^2+r^2]^{1/2}}+2\frac{\xi}{\sqrt{\xi^2+r^2}}\right] \end{aligned}\right\}. \tag{24}$$

3. Modified analytical solution by sub-models

3.1. General-segments of moving charges

The argument that in a closed segment, the field strength due to a uniformly moving charge is equal to the field strength due to a static charge is obvious only in the Maxwellian field theory. The reason for that is that the moving charges constitute a constant current that produces a constant magnetic field and therefore does not contribute a change in the electric field due to a constant charge density.

By using relativistic considerations and approach, the above-mentioned argument is not so obvious, because the evolution of the electric field strength, at the observation point $P(\xi, r)$ consists of several different kinds of fields, namely,

(1) a static field,

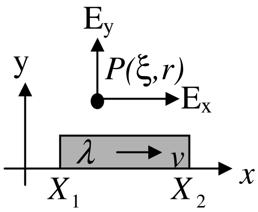

Figure 9. A closed segment of uniformly moving charges.

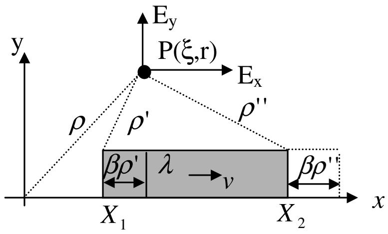

Figure 10. The evolution of the field from the observer's point of view.

(2) an acceleration field.

(3) a velocity field.

In a closed segment, the charges are stationery at point X_1 (see figure 9), then they are accelerated with an infinite acceleration to the velocity v, travelling to the other end of the segment X_2 and decelerate to zero velocity at X_2.

The calculation of the electric field strength values is done for the observation point $P(\xi, r)$. At this point, all the information arrives at a time delay, due the fact that the electric field strength expands from the charge to the observation point at the velocity of light. Therefore, seen from $P(\xi, r)$, the evolution of the field due to the segment of charges is as described in figure 10.

The calculation of the field at the observation point is constructed of the following contributions.

(1) *A static electric field strength at X_1, due to a point charge of magnitude of $\lambda\beta\rho''$.* The reason for this charge magnitude is that the information about the initial movement of the charge has to travel a distance of ρ' to the observation point $P(\xi, r)$. During this time the charge travels a distance of $\beta\rho'$. Therefore, all charges that are in between X_1 and $X_1 + \beta\rho'$ are contributing a field strength of a static charge at X_1. Thus, these charges can be looked upon as a static point charge located at X_1, with a magnitude of $\lambda\beta\rho'$.

The electric field strength values due to these charges in Cartesian coordinates are

$$\left.\begin{aligned} E_{xs1} &= \frac{\lambda\beta}{4\pi\varepsilon_0}\cdot\frac{\xi - X_1}{\rho'^2} \\ E_{ys1} &= \frac{\lambda\beta}{4\pi\varepsilon_0}\cdot\frac{r}{\rho'^2} \end{aligned}\right\}. \tag{25}$$

(2) *The 'acceleration' field of the charge density λ at point* X_1. The magnitude of this field is (from equation (22)):

$$\left.\begin{aligned} E_{xa2} &= -\frac{\lambda}{4\pi\varepsilon_0}\frac{\beta^2 r^2}{\rho'^2(\rho' - \beta(\xi - X_1))} \\ E_{ya2} &= \frac{\lambda}{4\pi\varepsilon_0}\frac{\beta^2 r(\xi - X_1)}{\rho'^2(\rho' - \beta(\xi - X_1))} \end{aligned}\right\}. \tag{26}$$

(3) *The 'velocity' field of the charge density of λ, in the interval of* $X_1 + \beta\rho' = x = X_2 + \beta\rho''$. The interval margins are justified by the fact that the information about the initial movement of the charge reaches the observation point at the same time when the charge has travelled a distance of $\beta\rho'$, as mentioned above. In the same manner at point X_2 the charge stops. Due to the fact that the information must travel a distance of ρ'' to the observation point $P(\xi, r)$. From the observer's point of view, the charge stopped at $X_2 + \beta\rho$ only. Therefore, the expressions for the electric field strengths, due to the 'velocity' field using the Special Relativity Theory is

$$\left.\begin{aligned} E_{xv3} &= \frac{\lambda}{4\pi\varepsilon_0}\int_{X_1+\beta\rho'}^{X_2+\beta\rho''} \mathrm{d}x \frac{\gamma(\xi - x)}{[\gamma^2(\xi - x)^2 + r^2]^{3/2}} \\ &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\left\{\frac{1}{\gamma}\cdot\left[\frac{1}{[\gamma^2(\xi - (X_2 + \beta\rho''))^2 + r^2]^{1/2}}\right]\right. \\ &\quad \left. -\frac{1}{\gamma}\cdot\left[\frac{1}{[\gamma^2(\xi - (X_1 + \beta\rho'))^2 + r^2]^{1/2}}\right]\right\} \\ E_{yv3} &= \frac{\lambda}{4\pi\varepsilon_0}\int_{X_1+\beta\rho'}^{X_2+\beta\rho''} \mathrm{d}x \frac{\gamma r}{[\gamma^2(\xi - x)^2 + r^2]^{3/2}} \\ &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{1}{r}\cdot\gamma\cdot\left[\frac{\xi - (X_2 + \beta\rho'')}{[\gamma^2(\xi - (X_2 + \beta\rho''))^2 + r^2]^{1/2}}\right. \\ &\quad \left. -\frac{\xi - (X_1 + \beta\rho')}{[\gamma^2(\xi - (X_1 + \beta\rho'))^2 + r^2]^{1/2}}\right] \end{aligned}\right\}. \tag{27}$$

(4) *An 'acceleration field' of a charge density of λ, accelerating at* X_2 *to a constant velocity of* v. When the charges reach point X_2 they stop. This can be described as the symmetrical case to the acceleration procedure at X_1. However, there is some difficulty to explain the physical reason for the charge's deceleration. The same results of calculating the electric field strength due to the decelerating charges to zero velocity at the point X_2, can be obtained by calculating the electric field strength due to accelerating negative charges, with charge density of $-\lambda$ at point X_2, in the opposite direction of the wave propagation. These charges are accelerated from a zero velocity to a constant velocity of v. The acceleration of the charges will start at time $t = (X_2 - X_1)/v$, after initiation. The electric field strength due to this 'acceleration' field is

$$\left.\begin{aligned} E_{xa4} &= \frac{\lambda}{4\pi\varepsilon_0}\frac{\beta^2 r^2}{\rho''^2[\rho'' - \beta(\xi - X_2)]} \\ E_{ya4} &= -\frac{\lambda}{4\pi\varepsilon_0}\frac{\beta^2 r(\xi - X_2)}{\rho''^2[\rho'' - \beta(\xi - X_2)]} \end{aligned}\right\} \tag{28}$$

where $\rho'' = \sqrt{(\xi - X_2)^2 + r^2}$.

(5) A static electric field strength at X_2, due to a point charge with the magnitude of $-\lambda\beta\rho''$. This field calculated as above (see field contribution 1) is

$$\left.\begin{aligned} E_{xs5} &= \frac{-\lambda\beta\rho''}{4\pi\varepsilon_0}\cdot\frac{(\xi - X_2)}{\rho''^3} = \frac{-\lambda\beta}{4\pi\varepsilon_0}\cdot\frac{(\xi - X_2)}{\rho''^2} \\ E_{ys5} &= \frac{-\lambda\beta\rho''}{4\pi\varepsilon_0}\cdot\frac{r}{\rho''^3} = \frac{-\lambda\beta}{4\pi\varepsilon_0}\cdot\frac{r}{\rho''^2} \end{aligned}\right\}. \tag{29}$$

The total electric field strength at the observation point $P(\xi, r)$, is the sum of the five fields contributions described above.

Thus the total field in the x-axis direction is

$$\begin{aligned} E_x &= \sum_{i=1}^{5} E_{x_i} = E_{x1} + E_{x2} + E_{x3} + E_{x4} + E_{x5} = \frac{\lambda}{4\pi\varepsilon_0}\cdot\left[\frac{1}{\rho''} - \frac{1}{\rho'}\right] \\ &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\left[\frac{1}{\sqrt{(\xi - X_2)^2 + r^2}} - \frac{1}{\sqrt{(\xi - X_1)^2 + r^2}}\right], \end{aligned} \tag{30}$$

and the field strength in the y-axis direction is

$$\begin{aligned} E_y &= \sum_{i=1}^{5} E_{y_i} = E_{y1} + E_{y2} + E_{y3} + E_{y4} + E_{y5} \\ &= \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{1}{r}\cdot\left[\frac{\xi - X_1}{\sqrt{(\xi - X_1)^2 + r^2}} - \frac{\xi - X_2}{\sqrt{(\xi - X_2)^2 + r^2}}\right]. \end{aligned} \tag{31}$$

The obtained result (see equations (30) and (31)) are consistent with the results obtained by calculating the electric field strengths due to a static charge density λ enclosed in the segment $[X_1, X_2]$. Therefore, the contribution of a segment of static charge is identical to the case that in that same segments the charges are travelling at constant velocity.

3.2. The sub-model

3.2.1. Graphical presentation. A step function can be observed as the case, in which charges are stationary at the origin and then at $t = 0$, they are accelerated at an infinite acceleration to a constant velocity of v. Thereafter, their velocity is constant. Due to this acceleration, the electric field strength at any observation point $P(\xi, r)$ cannot be determined by using the Special Relativity Theory only, as it deals with the cases, in which the charges move with a constant velocity only. Moreover, from the observer's point of view, one part of the current wave travels at a constant velocity v and the other part of the current wave is stationary at the origin. Furthermore, a solution by means of the Special Relativity Theory demands transformation of the problem to a different coordinate system, using Lorentz transformation. In this case, the source is stationary and the observation point can travel at a constant velocity. In the new coordinate system, the field strength is determined using Coulomb's law and then the coordinates are transformed back to the original system by the reverse Lorentz transformation. In the present case, the model has two current waves travelling in opposite direction. Therefore, it is impossible to find a coordinate system in which the conditions for using Special Relativity Theory are satisfied.

However in order to use the Special Relativity Theory, the model is assembled of sub-models in such a manner that all parts of the model are either stationary or travelling at a constant velocity. Thus, the electric field strength is calculated for each part (sub-model) for

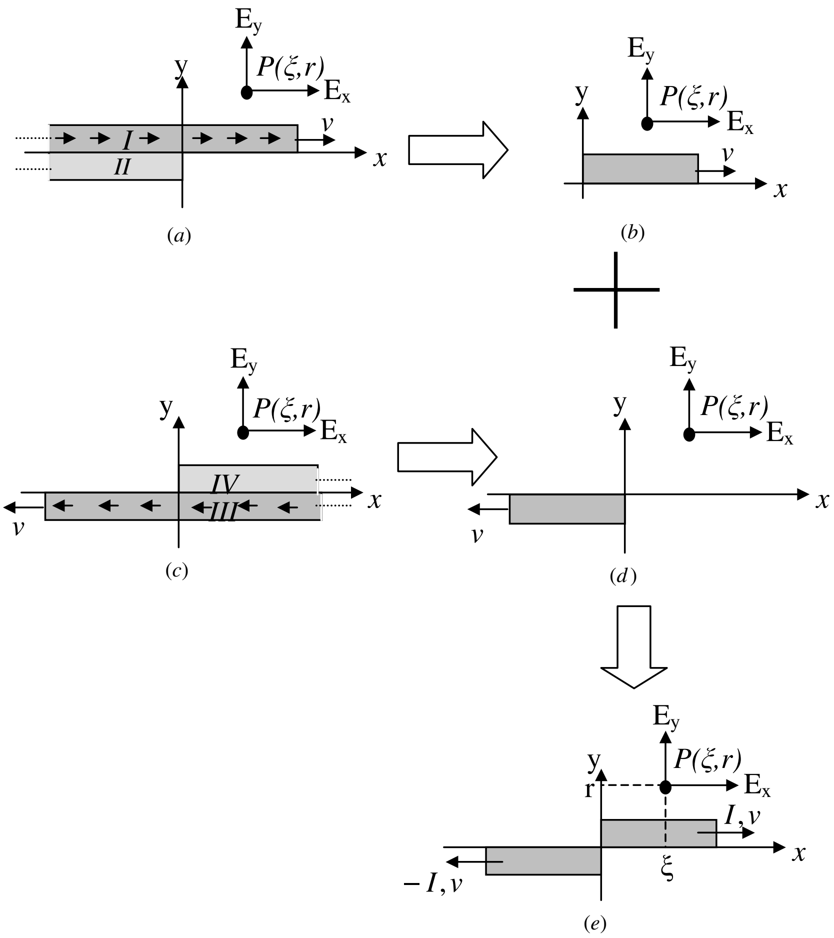

Figure 11. Assembly of the model by the different sub-models.

itself. Then by superposition theory the sum of the electric field strengths of all parts yields the desired solution. See figure 11.

In figure 11(a), the current wave, marked as *I*, consists of charges originating at infinity on the negative side of the *x*-axis. These charges move at a constant velocity v towards the positive direction of the *x*-axis as shown. Section 2 is a static segment of a charge density with an opposite polarity. This segment is located between infinity, on the negative side of the *x*-axis and the origin. The net sum of the electric field strengths at the observation point $P(\xi, r)$ is the same as the electric field strength at the same point, calculated by the current wave described in figure 11(*b*). The same situation is described in figures 11(*c*) and (*d*) for a current wave of opposite polarity, travelling in the opposite direction. The electric field strength due to the total model (see figure 11(*e*)) is the sum of the field strengths due to the current waves described in figures 11(*b*) and (*d*).

In figures 11(*a*) and (*c*) the influence of the current wave 'tail' is eliminated by adding the field strength due to the uniformly moving charges and the field strength due to the opposite polarity static charges. This operation has been justified, both physically and mathematically, in the last section.

3.2.2. Mathematical calculation. The wave-pair model is the step function response of a transient current wave. The model is constructed the by sub-models. Each sub-model consists of charges travelling at a constant velocity, v, only and segments of static charge densities. Therefore, the calculations of the electric field strength due to each part of the model are simplified. It is done either by the Special Relativity Theory for the travelling waves, or by calculating field strengths of static charge density distributions.

(1) The electric field strength of the wave travelling from $-\infty$ to X_1. In figure 11(*a*) the section marked as I is a current wave travelling at a constant velocity v. The calculation of the electric field at the observation point $P(\xi, r)$ is done, as before, by transforming the coordinates at hand into a new coordinate system. In this new system, the travelling wave is stationary and the observation point is travelling in the opposite direction, according to Lorentz transformation. Thus, the electric field strength is calculated by Coulomb's law and then it is transformed back by the inverse Lorentz transformation to the original coordinate system [15].
The resulting electric field strengths at the observation point $P(\xi, r)$ are

$$\left.\begin{aligned} \vec{E}_{XI} &= k\lambda \int_{-\infty}^{X_1} \frac{\gamma(\xi - x)\,\mathrm{d}x}{[\gamma^2(\xi - x)^2 + r^2]^{3/2}} = k\lambda \frac{1}{\gamma[\gamma^2(X_1 - \xi)^2 + r^2]^{1/2}} \\ \vec{E}_{YI} &= k\lambda \int_{-\infty}^{X_1} \frac{\gamma r\,\mathrm{d}x}{[\gamma^2(\xi - x)^2 + r^2]^{3/2}} = k\lambda \frac{\gamma(X_1 - \xi)}{r[\gamma^2(X_1 - \xi)^2 + r^2]^{1/2}} \end{aligned}\right\} . \tag{32}$$

(2) Cancellation of the wave's 'tail' from $-\infty$ to the origin. A calculation of the electric field strength at the observation point $P(\xi, r)$ due to a static charge density of $-\lambda$, located on the x-axis from $-\infty$ to the origin (see figure 11(*a*) II) is added to the total field strengths calculated by (32). This is done in order to cancel the influence of the electric field strength, due to the part of the travelling wave in the region of $-\infty$ to the origin. The solution yields the electric field strengths due to a current wave travelling in the positive direction of the x-axis, as shown in figure 11(*b*).
The electric field strengths due to the above-mentioned static charge density are

$$\left.\begin{aligned} \vec{E}_{XII} &= -k\lambda \int_{-\infty}^{0} \frac{(\xi - x)\,\mathrm{d}x}{[(\xi - x)^2 + r^2]^{3/2}} = -k\lambda \cdot \frac{1}{\sqrt{\xi^2 + r^2}} \\ \vec{E}_{YII} &= -k\lambda \int_{-\infty}^{0} \frac{r\,\mathrm{d}x}{[(\xi - x)^2 + r^2]^{3/2}} = -k\lambda \cdot \frac{1}{r} \cdot \frac{\xi}{\sqrt{\xi^2 + r^2}} \end{aligned}\right\} . \tag{33}$$

Thus, the total electric field strengths of the travelling current wave, see figure 11(*b*), are

$$\left.\begin{aligned} \vec{E}_X &= \vec{E}_{XI} + \vec{E}_{XII} = k\lambda \frac{1}{\gamma[\gamma^2(X_1 - \xi)^2 + r^2]^{1/2}} - k\lambda \frac{1}{\sqrt{\xi^2 + r^2}} \\ \vec{E}_Y &= \vec{E}_{YI} + \vec{E}_{YII} = k\lambda \frac{\gamma(X_1 - \xi)}{r[\gamma^2(X_1 - \xi)^2 + r^2]^{1/2}} + k\lambda \cdot \frac{1}{r} \cdot \frac{\xi}{\sqrt{\xi^2 + r^2}} \end{aligned}\right\} . \tag{34}$$

(3) The electric field strength of the total model. A similar calculation to the former one, demonstrated in the last section, can be done for the current wave travelling in the negative direction of the x-axis. The total electric field strengths due to the wave shown in figure 11(*d*) are

$$\left.\begin{aligned} \vec{E}_X &= \vec{E}_{XIII} + \vec{E}_{XIV} = k\lambda \frac{1}{\gamma[\gamma^2(X_1 + \xi)^2 + r^2]^{1/2}} - k\lambda \frac{1}{\sqrt{\xi^2 + r^2}} \\ \vec{E}_Y &= \vec{E}_{YIII} + \vec{E}_{YIV} = -k\lambda \frac{\gamma(X_1 + \xi)}{r[\gamma^2(X_1 + \xi)^2 + r^2]^{1/2}} + k\lambda \cdot \frac{1}{r} \cdot \frac{\xi}{\sqrt{\xi^2 + r^2}} \end{aligned}\right\} . \tag{35}$$

The solution for the total field strength of the model, presented in figure 11(e), is obtained by adding the field strength calculated at the observation point due to the positive travelling current wave, given by equation (34) and the field strength calculated for the negative charge density travelling in the negative direction of the x-axis, see equation (35). The total electric field strengths are

$$\left.\begin{aligned} \vec{E}_X &= k\lambda\left(\frac{1}{\gamma[\gamma^2(X_1-\xi)^2+r^2]^{1/2}}+\frac{1}{\gamma[\gamma^2(X_1+\xi)^2+r^2]^{1/2}}-\frac{2}{\sqrt{\xi^2+r^2}}\right)\\ \vec{E}_Y &= k\lambda\cdot\frac{1}{r}\left(\frac{\gamma(X_1-\xi)}{[\gamma^2(X_1-\xi)^2+r^2]^{1/2}}-\frac{\gamma(X_1+\xi)}{[\gamma^2(X_1+\xi)^2+r^2]^{1/2}}+\frac{2\xi}{\sqrt{\xi^2+r^2}}\right)\end{aligned}\right\}. \tag{36}$$

The obtained results (equation (36)) are identical with the electric field strengths calculated by using Maxwell's equations [14].

4. Conclusion

In this paper, the electric field due to a lightning strike is determined by using Special Relativity Theory in order to calculate the 'velocity field', and some relativistic concepts for calculating the 'acceleration field'. These fields are the basic elements required for calculating the total field resulting from the current wave-pair model or any other step wave model. The relativistic calculations are simpler than solving Maxwell's equations, due to the fact that in the Special Relativity Theory, the coordinates are transformed to a new coordinate system in which the source is stationary; and basically the calculations are performed by Coulomb's law.

The paper describes two analytical approaches to the problem.

(1) The direct approach-based on calculating the fields directly from the given model. In this approach, the two segments of the model are analysed separately. The field is calculated for each of the different stages of the evolution of the step wave in time. (static field, acceleration and moving at constant velocity). The net sum of all fields for the two segments gives the total field.

(2) *The modified sub-models approach*. The model is divided into sub-models. The sub-models consist of either segment of constant velocity moving charges, or segments of static charges. Each segment's electric field strength is calculated at the observation point $P(\xi, r)$, by means of Relativity Theory for the moving charges or Coulomb's law for the static charges. The superposition of all these fields yields the total electric field strength at the given observation point. A prove is given for justifying the operation of summing static and constant moving charge filaments.

(3) Although the wave-pair model was considered in this paper, we note that the relativistic approach could have been employed to other models mentioned in the introduction.

The results of both approaches are identical and been compared with the results obtained by using Maxwell's equations. The comparison yields identical results. Thus, in this paper, a novel, different and simpler and scientifically complimentary approach is presented. This relativistic approach gives solutions to the problem in a simple mathematical approach compared to solving Maxwell's equations which are partially differential equations. The presented approach can be applied to other models if they can be presented as a composition of step functions.

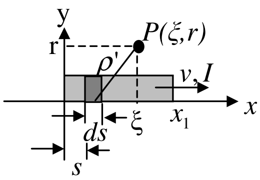

Figure A1. A current wave travelling to the right of the x-axis at a velocity of v.

Appendix

The method of calculating the induced electric fields due to lightning strike via relativity approach is compared with the results via Maxwell's equations. Therefore, a brief review of the calculation method through Maxwell's equations is presented in this appendix.

Maxwell's equations for free space are

$$\left.\begin{aligned} \operatorname{rot}\vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \operatorname{rot}\vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \\ \operatorname{div}\vec{D} &= \rho \\ \operatorname{div}\vec{B} &= 0 \end{aligned}\right\} \tag{A.1}$$

where the vector potential $\vec{A}$ is defined, based on the fourth term of equation (A.1):

$$\vec{B} = \operatorname{rot}\vec{A} \tag{A.2}$$

from equations (A.1) and (A.2):

$$\vec{E} = -\operatorname{grad} V - \frac{\partial \vec{A}}{\partial t}. \tag{A.3}$$

Based on the above-mentioned equations, together with Ohm's law, the wave equations for v and $\vec{A}$ can be obtained:

$$\left.\begin{aligned} \nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} &= -\frac{\rho}{\varepsilon_0} \\ \nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} &= -\mu_0 \vec{J} \end{aligned}\right\}. \tag{A.4}$$

These equations are valid only when the following boundary condition is fulfilled:

$$\operatorname{div}\vec{A} + \frac{1}{c^2}\frac{\partial V}{\partial t} = 0. \tag{A.5}$$

In a cylindrical system, where all conductors are usually thin, it is worthwhile to introduce λ, the charge per unit length, instead of ρ, and the current I, instead of $\vec{J}$. The general solutions for the scalar and vector potential equations, at point $P(\xi, r)$, which lies at a distance ρ' from an infinitesimal conductor element ds (see figure A1), are

$$\left.\begin{aligned} V &= \frac{1}{4\pi\varepsilon_0}\int_s \frac{\lambda\left(s, t - \frac{\rho'}{c}\right)}{\rho'} \cdot \mathrm{d}s \\ \vec{A} &= \frac{\mu_0}{4\pi}\int_s \frac{I\left(s, t - \frac{\rho'}{c}\right)}{\rho'} \cdot \mathrm{d}\vec{s} \end{aligned}\right\}. \tag{A.6}$$

These are the well-known 'retarded potentials'.

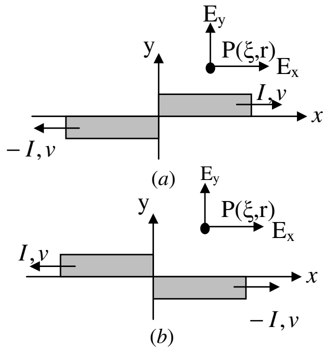

Figure A2. (*a*) An N–P wave-pair, (*b*) a P–N wave-pair.

Equation (A.6) is to be applied for the calculation of the electric field strength components at the point $P(\xi, r)$, due to a current wave travelling on the x-axis with the velocity of v, see figure A1. With the knowledge of the scalar and vector potentials, the field strength $\vec{E}$ can be obtained from equation (A.3). Prior to calculating $\vec{E}$, the vector and scalar potentials determined by equation (A.6) should be checked to fulfil the boundary condition of equation (A.5). In the case of a single current wave as described above, see figure A1, equation (A.5) is not satisfied. The reason for that is that the source of the current wave has not been taken into account. Thus, the physical picture is distorted.

Trying various types of wave models leads to the conclusion that the only topography of waves that satisfies the boundary condition of equation (A.5) is N–P or P–N wave-pairs, see figure A2.

An N–P wave-pair is a positive polarity current wave, which travels in the positive direction of the x-axis at a velocity of v, and a negative polarity current wave travelling in the opposite direction, see figure A2(*a*). A P–N wave-pair is the opposite case of an N–P wave-pair, see figure A2(*b*). In these cases, the source is physically represented.

Solving the potential equations for an N–P or a P–N model gives solutions which satisfy equation (A.5). These potentials are used in order to calculate the electric field strength $\vec{E}$ and the solutions are given in equation (24) in the paper.

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